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Chiral Symmetry Restoration at Finite Temperature in the Linear Sigma– Model

D. Metzger, H. Meyer–Ortmanns, H.–J. Pirner*

Institut für Theoret. Physik,
Universität Heidelberg

Abstract

The temperature behaviour of meson condensates $< \sigma_0 >$ and $< \sigma_8 >$ is calculated in the $SU(3) \times SU(3)$-linear sigma model. The couplings of the Lagrangian are fitted to the physical π, K, η, η' masses, the pion decay constant and a $O^+(I=0)$ scalar mass of $m_\sigma = 1.5$ GeV. The quartic terms of the mesonic interaction are converted to a quadratic term with the help of a Hubbard-Stratonovich transformation. Effective mass terms are generated this way, which are treated self-consistently to leading order of a $1/N$-expansion. We calculate the light $< \bar{q}q >$ and strange $< \bar{s}s >$-quark condensates using PCAC relations between the meson masses and condensates. For a cut-off value of 1.5 GeV we find a first-order chiral transition at a critical temperature $T_c \sim 161$ MeV. At this temperature the spontaneously broken subgroup $SU(2) \times SU(2)$ is restored. Entropy density, energy density and pressure are calculated for temperatures up to and slightly above the critical temperature. To our surprise we find some indications for a reduced contribution from strange mesons for $T \geq T_c$.

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1 Introduction

The study of finite temperature QCD is important both for theoretical and experimental hadron physics. Theoretically a hadron gas in thermal equilibrium is the simplest system to study the dynamics underlying deconfinement and chiral symmetry restoration. Experimentally a nearly equilibrated hadron gas with a transverse radius $R_\perp \approx 10$ fm is supposed to give the dominant contribution at a later stage of relativistic heavy ion collisions. In principle this allows an experimental check of the equation of state of hadrons. In the following work we approach the phase transition region from the low temperature side. Below $T \sim 100$ MeV pions are known to be the most relevant degrees of freedom. Above this temperature region also heavier hadrons give a non-negligible contribution to the condensates and thermodynamic quantities [1]. One way of including part of the heavier mesons is provided by the choice of $SU(3) \times SU(3)$ as chiral symmetry group rather than $SU(2) \times SU(2)$. The linear $SU(3) \times SU(3)$ sigma model includes a nonet of pseudoscalar (O^-)-fields and a nonet of scalar (O^+)-fields [2]. The spontaneous breaking of the $SU(3) \times SU(3)$ symmetry leads to massless (O^-) Goldstone modes. Obviously a massless pseudoscalar octet does not provide an adequate approximation to the experimentally observed meson spectrum. Therefore we include explicit symmetry breaking terms to account for the physical mass values of the octet-fields. A cubic term in the meson fields guarantees the correct mass splitting of the $\eta - \eta'$ masses which is due to the $U(1)$-anomaly. It reflects the 't Hooft-determinant on the quark level.

2 The model at zero temperature

For a Euclidean metric the Lagrangian of the linear sigma–model is given as

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial_\mu\Phi\partial_\mu\Phi^+ - \frac{1}{2}\mu_0^2\mathrm{tr}\Phi\Phi^+ + f_1\left(\mathrm{tr}\Phi\Phi^+\right)^2 + f_2\mathrm{tr}\left(\Phi\Phi^+\right)^2 \\
&\quad +g\left(\det\Phi + \det\Phi^+\right) + \varepsilon_0\sigma_0 + \varepsilon_8\sigma_8, \qquad (1)
\end{aligned}
$$

where the (3×3) matrix field $\Phi(x)$ is given in terms of Gell– Mann matrices λ_ℓ ($\ell = 0,\ldots,8$) as

$$
\Phi = \frac{1}{\sqrt{2}}\sum_{\ell=0}^{8}\left(\sigma_\ell + i\pi_\ell\right)\lambda_\ell. \qquad (2)
$$

Here σ_ℓ and π_ℓ denote the nonets of scalar and pseudoscalar mesons, respectively. As order parameters for the chiral transition we choose the meson condensates $<\sigma_0>$ and $<\sigma_8>$. The chiral symmetry of $\mathcal{L}$ is explicitly broken by the term $(-\varepsilon_0\sigma_0 - \varepsilon_8\sigma_8)$, corresponding to the finite quark mass term $m_q\bar{q}q + m_s\bar{s}s$ on the quark level. The chiral limit is realized for vanishing external fields ε_0 and ε_8. Note that the action $S = \int d^3x d\tau\mathcal{L}$ with $\mathcal{L}$ of Eq. (1) may be regarded as an effective action for QCD, constructed in terms of an order parameter field Φ for the chiral transition. It plays a similar role to Landau's free energy functional for a scalar order parameter field for investigating the phase structure of a Φ^4-theory. First conjectures about the chiral phase transition were based on a renormalization group analysis in momentum space [3, 4]. An ε-expansion in $d = 4-\varepsilon$ dimensions has been performed by Iacobson and Amit [4]. When it is applied to the Lagrangian of Eq. (1) with $g = 0, \varepsilon_0 = 0 = \varepsilon_8$, it predicts a first order chiral transition. For three flavours the det-term is cubic in the field components. Hence a non-vanishing g will further support the first order nature of the transition. In contrast finite mass terms ($\varepsilon_0 \neq 0 \neq \varepsilon_8$) may change the transition to a smooth crossover behaviour, if their values are large enough.

Patkós and Frei confirmed the first order nature of the chiral transition in the chiral limit of the $SU(3)\times SU(3)$ linear sigma model [5]. The result was obtained in a saddle point approximation to the free energy functional in three dimensions (dropping the imaginary time-dependence of the fields Φ in Eq. (1)). In a subsequent

work [6] it was shown that non-vanishing pseudoscalar meson masses actually change the first oder transition to a smooth crossover in the condensates, if otherwise the same approach is followed as in [5].

In the present work we have extended the former approach to a treatment of the full four-dimensional theory, keeping all Matsubara frequencies in the effective potential. The reason is that a complete dimensional reduction from 4 to 3 dimensions may be approximately realized at high temperatures or for a second order phase transition. In the present work we are interested in the low temperature region; we also cannot expect a second order transition in the presence of explicit symmetry breaking terms. Therefore contributions from non-zero Matsubara frequencies may even qualitatively change the results. This is in fact what we will demonstrate in this paper.

The six unknown couplings of the sigma-model (Eq. (1)) ($\mu_0^2, f_1, f_2, g, \varepsilon_0, \varepsilon_8$) are assumed to be temperature independent and fitted to the pseudoscalar masses at zero temperature. Further experimental input parameters are the pion decay constant $f_\pi = 93$ MeV and a high lying (O^+) scalar mass $m_\sigma = 1.59$ GeV (cf. Table 1). For the remaining scalar masses and the coupling constants we obtain the values of Table 1.

Input					
m_π [MeV]	$m_K[MeV]$	$m_\eta[MeV]$	$m_{\eta'}$ [MeV]	f_π [MeV]	$m_\sigma = m_{f_0}$ [MeV]
138.04	495.66	547.45	957.75	93	1590
Output					
μ_0^2 [GeV^2]	f_1	f_2	g [GeV]	ε_0 [GeV^3]	ε_8 [GeV^3]
0.758	12.166	3.053	1.527	0.02656	-0.03449
m_{a_0} [MeV]	$m_{K_0^*}$ [MeV]	$m_{f_0'}$ [MeV]	f_K [MeV]		
914.05	913.35	764.71	128.81		

Table 1: Tree level parametrization of the $SU(3) \times SU(3)$ linear sigma model (input data taken from experiment).

The interpretation of the observed scalar mesons is controversial. There are good reasons to interpret the (0^+) mesons at 980 MeV as meson bound states [7]. The

model underestimates the strange quark mass splitting in the scalar meson sector, the value for $m_{K_0^*}$ comes out too small.

The effective theory can be related to the underlying QCD Lagrangian by comparing the symmetry breaking terms in both Lagrangians and identifying terms with the same transformation behaviour under $SU(3) \times SU(3)$. Taking expectation values in these equations we obtain the following relations between the light quark condensates, strange quark condensates and meson condensates

$$
\begin{aligned}
< \bar{q}q > &= \frac{(-\varepsilon_0)}{2\hat{m} + m_s} < \sigma_0 > + \frac{(-\varepsilon_8)}{2(\hat{m} - m_s)} < \sigma_8 > \\
< \bar{s}s > &= \frac{(-\varepsilon_0)}{2\hat{m} + m_s} < \sigma_0 > - \frac{(-\varepsilon_8)}{(\hat{m} - m_s)} < \sigma_8 > .
\end{aligned} \tag{3}
$$

We use $\hat{m} \equiv (m_u + m_d)/2 = (11.25 \pm 1.45)$ MeV and $m_s = (205 \pm 50)$ MeV for the light and strange quark masses at a scale $\Lambda = 1$ GeV [8]. From the scalar meson condensates at $T = 0$, $\langle \sigma_0 \rangle = 0.144$ GeV and $\langle \sigma_8 \rangle = -0.0415$ GeV we get

$$
\begin{aligned}
\langle \bar{q}q \rangle &= -(235 \pm 60\mathrm{MeV})^3 \\
\langle \bar{s}s \rangle &= -(290 \pm 30\mathrm{MeV})^3
\end{aligned} \tag{4}
$$

in accordance with values from PCAC relations [8] within the error bars. Since we treat the coefficients $\varepsilon_0, \varepsilon_8$ of $< \sigma_0 >$ and $< \sigma_8 >$, and $\hat{m}, m_s$ of $< \bar{q}q >$ and $< \bar{s}s >$ as temperature independent, we will use Eqs. (3) for all temperatures to translate our results for meson condensates into quark condensates.

We also check that the pseudoscalar meson mass squares, in particular m_π^2 and m_K^2 are linear functions of the symmetry breaking parameters $\varepsilon_0, \varepsilon_8$. Varying $\varepsilon_0, \varepsilon_8$ while keeping the other couplings fixed we can simulate the sigma model at unphysical meson masses. Since the current quark masses are assumed to depend linearly on ε_0 and ε_8, an arbitrary meson mass set can be related to a mass point in the $(m_{u,d}, m_s)$-plane by specifying the choice of $(\varepsilon_0, \varepsilon_8)$. This may be useful in order to

compare our results for meson (and quark) condensates with lattice simulations of the chiral transition.

3 Thermodynamics

The thermodynamics of the linear sigma model is determined by the partition function with the Lagrangian of Eq. (1)

$$Z = \int \mathcal{D}\Phi \exp\left\{ - \int_0^\beta d\tau \int d^3x \mathcal{L}(\Phi(\vec{x}, \tau)) \right\}. \tag{5}$$

We will treat Z in a saddle point approximation. The saddle point approximation amounts to the leading order of a $1/N$-expansion in this model, where $N = 2N_f^2 = 18$. Note that $\mathcal{L}$ of Eq. (1) would be $O(N)$- invariant, if $f_2 = 0$ and $g = 0$. Our input parameters lead to non-vanishing values of f_2 and g, therefore the $O(N)$-symmetry is only approximately realized.

We calculate the effective potential as a constrained free energy density $U_{eff}(\xi_0, \xi_8)$, that is the free energy density of the system under the constraint that the average values of σ_0 and σ_8 take some prescribed values ξ_0 and ξ_8. The values $\xi_{0_{min}}$ and $\xi_{8_{min}}$ that minimize U_{eff}, give the physically relevant, temperature dependent vacuum expectation values, i.e. $< \sigma_0 >= \xi_{0_{min}}$, $< \sigma_8 >= \xi_{8_{min}}$. Hence we start with the background field ansatz

$$\begin{aligned} \sigma_0 &= \xi_0 + \sigma_0' \\ \sigma_8 &= \xi_8 + \sigma_8', \end{aligned} \tag{6}$$

where σ_0' and σ_8' denote the fluctuations around the classical background fields ξ_0 and ξ_8. All other field components are assumed to have zero vacuum expectation value, i.e. $\sigma_\ell = \sigma_\ell'$ for $\ell = 1, \ldots, 7$ and $\pi_\ell = \pi_\ell'$ for $\ell = 0, \ldots, 8$. The relation between the effective potential U_{eff} and Z is given by

$$
\begin{aligned}
Z &= \int d\xi_0 \int d\xi_8 \hat{Z}(\xi_0, \xi_8) \\
\hat{Z}(\xi_0, \xi_8) &= : e^{-\beta V U_{eff}(\xi_0, \xi_8)} \\
&= \int \mathcal{D}\Phi \delta \left[\int \sigma_0(\vec{x}, \tau) - \xi_0 \right] \delta \left[\int \sigma_8(\vec{x}, \tau) - \xi_8 \right] \\
&\quad \cdot \prod_{\ell \neq 0,8} \delta \left[\int \sigma_\ell(\vec{x}, \tau) \right] \prod_{\ell=0}^{8} \delta \left[\int \pi_\ell(\vec{x}, \tau) \right] \cdot e^{-\int_0^\beta d\tau \int d^3x \mathcal{L}[\Phi]}
\end{aligned} \tag{7}
$$

where $' \int '$ is a short-hand notation for $\frac{1}{\beta V} \int_0^\beta d\tau \int d^3x$.

Next we insert the background field ansatz (6) in $\mathcal{L}$ and expand the Lagrangian in powers of $\Phi' = \frac{1}{\sqrt{2}} \sum_{\ell=0}^{8} (\sigma'_\ell + i\pi'_\ell)\lambda_\ell$. The constant terms in Φ' lead to the classical part of the effective potential U_{class}

$$
\begin{aligned}
U_{class}(\xi_0, \xi_8) &= -\frac{1}{2}\mu_0^2 \left(\xi_0^2 + \xi_8^2 \right) + \frac{g}{3\sqrt{3}} \cdot \left(2\xi_0^3 - \sqrt{2}\xi_8^3 - 3\xi_0\xi_8^2 \right) - \frac{2\sqrt{2}}{3} f_2 \xi_0 \xi_8^3 \\
&\quad + \left(f_1 + \frac{f_2}{3} \right) \xi_0^4 + \left(f_1 + \frac{f_2}{2} \right) \xi_8^4 + 2\left(f_1 + f_2 \right) \xi_0^2 \xi_8^2 - \varepsilon_0 \xi_0 - \varepsilon_8 \xi_8 .
\end{aligned} \tag{8}
$$

Linear terms in Φ'_ℓ vanish for all $\ell = 0, \ldots, 8$ due to the δ-constraints in Eq. (7).

Quadratic terms in Φ' define the isospin multiplet masses m_Q^2, where $Q = 1, \ldots, 8$ labels the multiplets. The contribution to the Lagrangian is denoted by $\mathcal{L}^{(2)}$

$$
\mathcal{L}^{(2)} = \frac{1}{2} \sum_Q g(Q) \left(\partial_\mu \varphi'_Q \partial_\mu \varphi'^{\dagger}_Q + m_Q^2 \varphi'_Q \varphi'^{\dagger}_Q \right). \tag{9}
$$

Here φ'_Q denotes σ'_Q for $Q = 1, \ldots, 4$ and π'_Q for $Q = 5, \ldots, 8$, $g(Q)$ is the multiplicity of the isospin multiplet. We have $g(1) = 3$ for the pions, $g(2) = 4$ for the kaons, $g(3) = 1 = g(4)$ for η, η', respectively. Correspondingly, the multiplicities for the scalar nonets are $g(5) = 3$, $g(6) = 4$, $g(7) = 1$, $g(8) = 1$ for the a_0, K_0^*, f_0, f'_0-mesons. Typical expressions for the masses are the pseudoscalar and scalar triplet masses

$$\begin{aligned} m_\pi^2 &= -\mu_0^2 + \left(4f_1 + \frac{4}{3}f_2\right)\xi_0^2 + \left(4f_1 + \frac{2}{3}f_2\right)\xi_8^2 \\ &\quad + \frac{4}{3}\sqrt{2}f_2\xi_0\xi_8 + \frac{2}{\sqrt{3}}g\xi_0 - 2\sqrt{\frac{2}{3}}g\xi_8 \qquad (10) \\ m_{a_0}^2 &= -\mu_0^2 + (4f_1 + 4f_2)\,\xi_0^2 + (4f_1 + 2f_2)\,\xi_8^2 \\ &\quad + \frac{8}{\sqrt{2}}f_2\xi_0\xi_8 - \frac{2}{\sqrt{3}}g\xi_0 + 2\sqrt{\frac{2}{3}}g\xi_8. \qquad (11) \end{aligned}$$

The cubic part in Φ' will be neglected, while the quartic term $\mathcal{L}^{(4)}$

$$\mathcal{L}^{(4)} = f_1(\mathrm{tr}\Phi'\Phi'^\dagger)^2 + f_2\mathrm{tr}(\Phi'\Phi'^\dagger)^2 \qquad (12)$$

is quadratized by introducing an auxiliary matrix field $\sum(\vec{x},\tau)$. This is a matrix version of a Hubbard-Stratonovich transformation [9]. We have the identity

$$\begin{aligned} & \exp\left\{\int_0^\beta d\tau \int d^3x(-)\left[f_1\left(\mathrm{tr}\,\Phi'\Phi'^\dagger\right)^2 + f_2\mathrm{tr}\,\left(\Phi'\Phi'^\dagger\right)^2\right]\right\} \\ = \;& \mathrm{const}\cdot\int_{c-i\infty}^{c+i\infty}\mathcal{D}\Sigma\,\exp\left[\frac{1}{16(\varepsilon+3\alpha)^2}\int_0^\beta d\tau\int d^3x\right. \\ & \left\{\mathrm{tr}\,\Sigma^2 + 2\mu_0^2\mathrm{tr}\,\Sigma - 8(\varepsilon+3\alpha)[\varepsilon\mathrm{tr}\,\left(\Sigma\Phi'\Phi'^\dagger\right) + \right. \\ & \left.\left. +\alpha\mathrm{tr}\,\Sigma\cdot\mathrm{tr}\,\left(\Phi'\Phi'^\dagger\right) + \mu_0^2\varepsilon\mathrm{tr}(\Phi'\Phi'^\dagger) + 3\alpha\mu_0^2\mathrm{tr}\,(\Phi'\Phi'^\dagger)]\right\}\right], \qquad (13) \end{aligned}$$

where

$$\begin{aligned} 2\varepsilon\alpha + 3\alpha^2 &\equiv f_1 \\ \varepsilon^2 &\equiv f_2 \quad . \qquad (14) \end{aligned}$$

In the saddle point approximation we drop $\int\mathcal{D}\Sigma$. As $SU(3)$- symmetric ansatz we use a constant diagonal matrix

$$\Sigma = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix} \quad . \qquad (15)$$

The choice of s will be optimized later, cf. Eq. (28). When Z is rewritten upon using Eq. (13), $\mathcal{L}^{(4)}$ of Eq. (12) is replaced by $\mathcal{L}^{(4)\prime}$ given as

$$\mathcal{L}^{(4)\prime} = -\frac{3}{8(3f_1+f_2)}\left(\frac{s^2}{2}+\mu_0^2 s\right)+\frac{1}{2}\left(s+\mu_0^2\right)\ \mathrm{tr}\,(\Phi'\Phi'^{\dagger}). \tag{16}$$

Hence the effect of the quadratization procedure is to induce an extra mass term $(s+\mu_0^2)$ and a contribution U_{saddle} to U_{eff}, which is independent of ξ_0 and ξ_8.

$$U_{\text{saddle}} = -\frac{3}{8(3f_1+f_2)}\left(\frac{s^2}{2}+\mu_0^2 s\right). \tag{17}$$

We are not aware of an analogous identity to Eq. (13), which includes the det-term of $\mathcal{L}$ and leads to a tractable form. Therefore we drop the cubic term in Φ' as mentioned above.

This way we finally end up with the following expression for $\hat{Z}$

$$\begin{aligned}\hat{Z}(\xi_0,\xi_8) \quad &= \quad e^{-\beta V(U_{\text{class}}+U_{\text{saddle}})}\ \cdot \\ &\cdot \int \prod_{Q=1}^{8} \mathcal{D}\varphi'_Q e^{-\int_0^\beta d\tau \int d^3x \frac{1}{2}\sum_Q g(Q)(\partial_\mu\varphi'_Q\partial_\mu\varphi'^{\dagger}_Q + X_Q^2\varphi'_Q\varphi'^{\dagger}_Q)}\end{aligned} \tag{18}$$

where

$$X_Q^2 \equiv m_Q^2+\mu_0^2+s. \tag{19}$$

Thus we are left with an effectively free field theory. The only remnant of the interaction appears in the effective mass squared X_Q^2 via the auxiliary field s.

The choice of a self-consistent effective meson mass squared has been pursued already in Refs. [5, 6]. This is an essentially new ingredient compared to earlier calculations of the chiral transition in the linear sigma model [10]. The positive contribution of s to the effective mass extends the temperature region, where imaginary parts in the effective potential can be avoided. In general, imaginary parts are encountered, when the effective mass-arguments of logarithmic terms become negative. They are an artifact of the perturbative evaluation of the effective potential and of no physical significance, as long as the volume is infinite. In our application the optimized choice for s will increase as function of temperature and lead to positive X_Q^2 over a wide range of parameters.

Gaussian integration over the fluctuating fields Φ' in Eq. (18) gives

$$\hat{Z}(\xi_0,\xi_8) = \exp\{-\beta V [U_{\rm class} + U_{\rm saddle} + \frac{1}{2\beta}\sum_{Q=1}^{8} g(Q)\sum_{n\in\not{Z}}\int\frac{d^3k}{(2\pi)^3}\,\ln\left(\beta^2(\omega_n^2+\omega_Q^2)\right)]\} \tag{20}$$

where

$$\omega_Q^2 \equiv k^2 + X_Q^2, \tag{21}$$

and

$$\omega_n^2 \equiv (2\pi n/T)^2 \tag{22}$$

denote the Matsubara frequencies. In contrast to our former approach [6] we keep all Matsubara frequencies and evaluate $\sum_{n\in\not{Z}}$ in the standard way, see e.g. [11]. The result is

$$\hat{Z}(\xi_0,\xi_8;s) = e^{-\beta V U_{\rm eff}(\xi_0,\xi_8;s)} \tag{23}$$

$$U_{\rm eff}(\xi_0,\xi_8;s) = U_{\rm class} + U_{\rm saddle} + U_{\rm th} + U_0 \tag{24}$$

$$U_{\rm th} \equiv \frac{1}{\beta}\sum_{Q=1}^{8} g(Q)\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\beta\omega_Q}\right) \tag{25}$$

$$U_0 \equiv \frac{1}{2}\sum_{Q=1}^{8} g(Q)\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\omega_Q. \tag{26}$$

Here we have indicated that $\hat{Z}$ and $U_{\rm eff}$ still depend explicitly on the auxiliary field s. The integral in Eq. (26) is regularized with a three-momentum cut-off Λ. The thermal contribution $U_{\rm th}$ vanishes at zero temperature and is finite for $T > 0$, while the zero point energy U_0 diverges as $\Lambda \to \infty$.

The linear sigma model is a renormalizable theory, and the cut-off could be removed after a suitable renormalization prescription. Since we are dealing with an effective model, the need for such a renormalization may be less obvious. Anyway we do not believe in this model as an effective description for QCD, when the momenta exceed a certain scale, say $\Lambda \approx 1 - 1.5$ GeV. The necessity for a renormalization arises, when we postulate a matching between the physical masses and condensates

with the $T = 0$-values, and T approaches zero from above. Such a matching is guaranteed, if we impose the following subtractions on the zero point energy part

$$
\begin{aligned}
&U_0^{\rm ren}(X_Q^2(\xi_0,\xi_8);\Lambda) := U_0(X_Q^2) - \{U_0(m_{\rm phys}^2) + \\
&\frac{\partial U_0(m^2)}{\partial m^2}|_{m_{\rm phys}^2} \cdot (X_Q^2 - m_{\rm phys}^2) + \frac{1}{2}\frac{\partial^2 U_0(m^2)}{\partial (m^2)^2}|_{m_{\rm phys}^2} \cdot (X_Q^2 - m_{\rm phys}^2)^2\}. \quad (27)
\end{aligned}
$$

Here $m_{\rm phys}^2$ is given by m_Q^2 of Eq. (9) evaluated at $\xi_0 =< \xi_0 >$ and $\xi_8 =< \xi_8 >$, i.e. for physical condensate values. The optimal choice s^* for the auxiliary field s is then determined by

$$
\frac{\partial U_{\rm eff}^{\rm ren}}{\partial s}|_{s^*} = 0, \quad (28)
$$

where $U_{\rm eff}^{\rm ren}$ equals $U_{\rm eff}$ of Eq. (24) with U_0 replaced by $U_0^{\rm ren}$ of Eq. (27).

Upon using Eq. (28) it is easily verified that $< \xi_i >$, defined as

$$
< \xi_i >= \frac{1}{\beta V}\frac{\partial \ln Z}{\partial \varepsilon_i}, \quad i = 0, 8, \quad (29)
$$

is free of extra contributions from the zero point energies at $T = 0$ if $\ln Z = -\beta V U_{\rm eff}^{\rm ren}(\xi_0, \xi_8; s^*)$. Thus a matching with $< \xi_i >_T = 0$ is ensured. Similarly we find for the effective masses

$$
X_Q^2|_{T=0} = m_Q^2 + s + \mu_0^2 = m_{\rm phys}^2 \quad (30)
$$

for $\xi_0 =< \xi_0 >$, $\xi_8 =< \xi_8 >$, since $s = -\mu_0^2$ at $T = 0$.

Note that the sensitivity to the cut-off in Eq. (27) is reduced from a Λ^4-dependence to a $1/\Lambda^2$-dependence. This is a desirable feature in view of the uncertainties in a suitable choice for Λ. We have taken $\Lambda = 1.5$ GeV and kept the cut-off finite throughout the calculations.

A further argument in favour of keeping the cut-off finite relies on results of Bardeen and Moshe [12] on the $1/N$-expansion in $O(N)$-theories. According to these results a symmetry restored groundstate occurs as vacuum state at zero temperature, if the cut-off is sent to infinity.

Now we are prepared to determine the temperature dependence of the order parameters $<\xi_0>(T)$, $<\xi_8>(T)$ from the minima of $U_{\rm eff}^{\rm ren}(\xi_0,\xi_8;s^*)$. Thermodynamic quantities like energy densities, entropy densities and pressure can be derived from Z in the standard way, if Z is approximated as

$$\hat{Z}^{\rm ren} \equiv e^{-\beta V U_{\rm eff}^{\rm ren}(\xi_0,\xi_8;s^*)}. \tag{31}$$

4 Results

For the parameters of Table 1 we vary the temperature and determine for each T the extremum of $U_{\rm eff}$ as a function of ξ_0, ξ_8 and s. The extremum is a minimum with respect to ξ_0 and ξ_8 and a maximum with respect to s. (For zero temperature this is easily seen from the explicit form of $U_{\rm class}$ (Eq. (8)) and $U_{\rm saddle}$ (Eq. (17)). Numerically the most convenient procedure is to determine the common zeroes in the derivatives $\frac{\partial U_{\rm eff}}{\partial \xi_0}$, $\frac{\partial U_{\rm eff}}{\partial \xi_8}$ and $\frac{\partial U_{\rm eff}}{\partial s}$ first, and then to check the desired minimum/maximum properties of the saddle point.

In Figs. 1 and 2 we show the variations of $\frac{<\bar{q}q>(T)}{<\bar{q}q>T=0}$ and $\frac{<\bar{s}s>(T)}{<\bar{s}s>T=0}$ as a function of temperature obtained from $<\xi_0>(T)$ and $<\xi_8>(T)$ with the help of Eq. (3). We observe a gradual decrease of the light quark condensate, whereas the strange quark condensate stays remarkably constant. A first order transition occurs at $T_c = 161$ MeV for $\Lambda = 1.5$ GeV. Note that we have still a cubic term in the classical part of the potential (Eq. (8)), the term we have dropped is cubic in the fluctuating fields. The critical temperature is determined in such a way that the pressure is continuous at T_c as a function of temperature. The values for the mesonic condensates show pronounced hysteresis effects, which are characteristic for a first order phase transition.

At T_c the strange quark condensate does not drop to zero. Only the $SU(2)\times SU(2)$ part of the chiral symmetry is restored within numerical errors. While $<\sigma_0>, <\sigma_8>$ have numerical errors $\Delta <\sigma> / <\sigma> = 0.1\%$, the error on the $<\bar{q}q>$ condensate is $\frac{\Delta<\bar{q}q>}{<\bar{q}q>} = 4\%$. Systematic errors are attached to the

current quark masses. These errors give the main contribution to $\frac{\Delta<\bar{q}q>}{<\bar{q}q>} = 25\%$. An exact restoration cannot be expected, since the symmetry is explicitly broken in the Lagrangian. When we derive the quark condensates from Eq. (3), we treat the quark masses and the external fields as temperature independent up to the transition region. This assumption may not be justified. The corresponding error is unknown.

In our lowest order calculation of the effective potential we cannot distinguish between pole- and screening masses. By 'masses' we mean the effective masses X_Q^2 entering the arguments of the logarithm according to Eq. (19), evaluated at the physical mass point $\xi_0 =< \xi_0 >$ and $\xi_8 =< \xi_8 >$, cf. e.g. Eqs. (10)-(11). Thus the temperature dependence of these masses is determined by the temperature dependence of the condensates. The masses m_π and $m_{f_0'}$ become degenerate for temperatures $T \geq T_c$ within the errors $m_\pi \approx (139 \pm 10$ MeV), $m_{f_0'} \approx (141 \pm 10$ MeV) at T_c, when T_c is approached from above. The degeneracy is a result of the vanishing light quark condensate. A zero value of $< \bar{q}q >$ above T_c implies a relation between $< \sigma_0 >$ and $< \sigma_8 >$, which leads to $m_\pi^2 \approx m_{f_0'}^2$. Below T_c the meson masses stay remarkably constant. The pion mass m_π and the scalar mass $m_{f_0'}$ change from $m_\pi = (150 \pm 10$ MeV) and $m_{f_0'} = (625 \pm 10$ MeV) at $T = T_c$ approaching T_c from below to $m_\pi = (139 \pm 10$ MeV) and $m_{f_0'} = (141 \pm 10$ MeV) at $T = T_c$ =161 MeV approaching T_c from above.

The behaviour of the scalar mass can induce an abrupt change in temperature dependent cross sections in contrast to a smooth variation in case of a second order transition or a crossover phenomenon. Temperature dependent cross sections may be realized in heavy ion collisions, when the transient quark gluon plasma cools down to the hadron phase and the hadron phase evolves until freeze-out.

The f_0' above T_c can no longer decay into two pions, its width for $T > T_c$ has a contribution from $f_0' \to 2\gamma$ decays with an invariant mass $m^2(2\gamma) \approx m_{\pi^0}^2$. In the experiment one may see an anomalous amount of such (2γ) decays, when the system spends a sizeable time in the phase where chiral symmetry is restored. In

the experiment WA98 at CERN a measurement of the number of gammas to the number of charged mesons, i.e. mostly $\pi^{\pm}$, is planned. Above T_c, the $\pi - K$ splitting is increased rather than reduced. The mass difference of $\Delta m = m_K - m_\pi = (400 \pm 20$ MeV) below T_c ($T < 161$ MeV) is increased to $\Delta m = (516 \pm 2$ MeV) for $T = 161$ MeV $\geq T_c$. Accordingly the strange meson contribution to the energy density in this temperature region is reduced compared to the low-temperature hadron gas.

In Fig. 3 we give the energy density ε/T^4 and pressure p/T^4 as function of temperature for a cut-off $\Lambda = 1.5$ GeV. The gap in the energy densities at T_c, which is a measure for the latent heat, is obviously rather small, about 10% of ε at T_c. Sizeable contributions to ε come mainly from 8 degrees of freedom, the pions, the kaons and the f_0 meson. The small value of the latent heat may be due to the vicinity of a (hypothetical) first order boundary in the (m_K, m_π)-mass diagram. At the location of this boundary the meson masses become so large that they wash out the chiral transition completely. We plan to check this explanation by investigating the chiral transition for unphysical values of the strange quark and light quark masses in the future.

The existence of quasibound $\sigma(f_0')$ and π modes may be a good approximation in the vicinity of T_c. Far above T_c the linear sigma model certainly fails as an effective model for QCD due to the lack of quark-gluon degrees of freedom. Nevertheless it would be interesting to study, at what temperature the full $SU(3) \times SU(3)$ symmetry is restored. At high temperatures the effective potential becomes proportional to $\sum_Q X^2(Q)T^2$, the linear terms in the meson masses (Eqs. (10)-(11)) cancel and temperature tries to fully restore the broken symmetry.

Finally we remark that our value for T_c is rather close to the lower limit of the Hagedorn temperature $T_H (T_H \sim 160$ MeV) [13]. This may not be entirely accidental. In our $1/N$-expansion N means a large number of flavours, since

$$N = 2 \cdot N_f^2. \tag{32}$$

In order to keep QCD an asymptotically free theory also the number of colours

N_c has to increase. Correspondingly our approximation is similar to Hagedorn's description of the hadron gas as a resonance gas. We expect that corrections from subleading terms in our $1/N_f$-expansion will implicitly amount to corrections also to the large N_c-limit.

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Figure Captions

Figure 1 Normalized light quark condensate $\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{T=0}}$ vs temperature.

Figure 2 Normalized strange quark condensate $\frac{\langle \bar{s}s \rangle}{\langle \bar{s}s \rangle_{T=0}}$ vs temperature.

Figure 3 Normalized energy density $\frac{\varepsilon}{T^4}$ and pressure $\frac{p}{T^4}$ vs temperature. The decrease of these quantities above $T \approx 200$ MeV as a function of T indicates the breakdown of our approximation scheme.